Name__________________

Polynomials Study Guide

Polynomial—a sum or difference of many terms

Example: $3x^2 + 15$

Terms—parts of a polynomial, each separated by a plus or minus sign

Example: $3x^2$ or 15

Degree—the sum of the exponents of variables in a term

Example: $3x^2$ is second degree, 15 is zero[th] degree,
and $4a^2b$ is third degree

Degree of a Polynomial—the highest degree of any term in that polynomial

Example: $3x^2 +15$ is second degree

Standard form of a Polynomial—form in which the degree of each term decreases from left to right

Example: $15 + 3x^2$ is *not* in standard form, but $3x^2 + 15$ is.

Below are two charts showing the names of polynomials with specific degrees or numbers of terms.

Degree	***Name***	***Example***
0	Constant	15
1	Linear	$4x - 7$
2	Quadratic	$3x^2 + 15$
3	Cubic	$x^3 + x$
4 (or higher)	4[th] degree (or whatever)	x^4

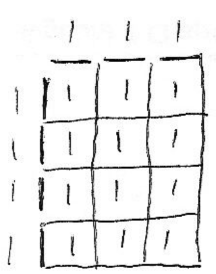
Number of Terms	***Name***
1	Monomial
2	Binomial
3	Trinomial
2 or more	Polynomial

There are six different methods we've learned to multiply polynomials.

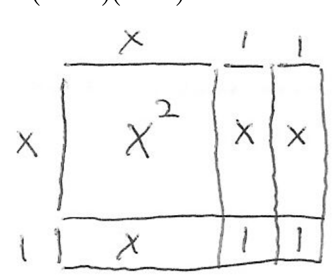
Area Method

The first method we learned is the area method. To multiply $3 \cdot 4$, you can draw a rectangle like the one below, 3 units by 4 units, then count the area of the rectangle. Likewise, we can multiply $(x+2)(x+1)$ by making a rectangle with sides of x+2 units and x+1 units, then count the area.

$3 \cdot 4 = 12$



$(x+2)(x+1) = x^2 + 3x + 2$



Distributive Method
Just like using the Distributive Property, we can split up multiplied polynomials using the distributive method, as shown below. In each case, one of the polynomials is split up and distributed, but still multiplied by the other polynomial. Then the distributive property is used and the expression simplified.

$$\begin{aligned}&(2x+3)(4x-2)\\&=2x(4x-2)+3(4x-2)\\&=8x^2-4x+12x-6\\&=8x^2+8x-6\end{aligned}$$

OR

$$\begin{aligned}&(2x+3)(4x-2)\\&=4x(2x+3)-2(2x+3)\\&=8x^2+12x-4x-6\\&=8x^2+8x-6\end{aligned}$$

FOIL Method
FOIL stands for First, Outside, Inside, and Last. These are descriptions of which terms to multiply when you multiply two binomials.

$$(2x+3)(4x-2)$$

First: $2x\cdot 4x=8x^2$
Outside: $2x\cdot -2=-4x$
Inside: $3\cdot 4x=12x$
Last: $3\cdot -2=-6$

$$(2x+3)(4x-2)=8x^2-4x+12x-6=8x^2+8x-6$$

Horizontal Method
The horizontal method is just like the FOIL method, except you can use it with more than just binomials. Make sure each term in the first polynomial is multiplied by each term in the second polynomial.

$$(2x+3)(x^2+4x-2)$$

$2x\cdot x^2=2x^3$ | $3\cdot x^2=3x^2$
$2x\cdot 4x=8x^2$ | $3\cdot 4x=12x$
$2x\cdot -2=-4x$ | $3\cdot -2=-6$

$$\begin{aligned}&(2x+3)(x^2+4x-2)\\&=2x^3+8x^2-4x+3x^2+12x-6\\&=2x^3+11x^2+8x-6\end{aligned}$$

Vertical Method

The vertical method is just like the standard way of multiplying large numbers on paper. Instead of each column representing a place value, however, each column represents a degree of term. The right-most column represents constants, and each column you move to the left increases in degree by one.

$$
\begin{array}{r}
3x^2 + 0x + 5 \\
\cdot \quad 2x - 7 \\
\hline
-21x^2 + 0x - 35 \\
6x^3 + 0x^2 + 10x + 0 \\
\hline
6x^3 - 21x^2 + 10x - 35
\end{array}
$$

Box Method

The box method involves drawing a box with four squares inside, then writing the two binomials, one on the top of the box, and one on the side. Next, for each square, multiply the term on top by the term on the side. Finally, simplify to get your answer.

	2x	5
x	$2x^2$	5x
-4	-8x	-20

$$(2x+5)(x-4)$$
$$= 2x^2 + 5x - 8x - 20$$
$$= 2x^2 - 3x - 20$$

Factoring—the process of expressing a polynomial as a product of its factors, similar to the opposite of multiplying polynomials

GCF Method

This method factors out a single term, the greatest common factor, from a polynomial. First, find the GCF of the terms in a polynomial. Then, divide each term by the GCF. Finally, express the polynomial as a product of the GCF and the divided terms.

$$2x^3 - 10x^2 + 8x$$
$$2x^3 = 2 \cdot x \cdot x \cdot x$$
$$10x^2 = 2 \cdot 5 \cdot x \cdot x$$
$$8x = 2 \cdot 2 \cdot 2 \cdot x$$
$$GCF = 2 \cdot x = 2x$$

$$\frac{2x^3}{2x} = x^2$$
$$\frac{-10x^2}{2x} = -5x$$
$$\frac{8x}{2x} = 4$$

$$2x^3 - 10x^2 + 8x = 2x(x^2 - 5x + 4)$$

Sum & Product Method
Alright, this is the trickiest method of all because it is used in slightly different ways depending on the situation. This method is used to factor quadratic trinomials in the form $ax^2 + bx + c$. The sum is always equal to b, and the product is equal to $a \cdot c$. The goal is to find two numbers that have this sum and product

In the case that a = 1, the polynomial can be factored into $(x + n_1)(x + n_2)$, where n_1 and n_2 are the two numbers that have the given sum and product.

$$x^2 - 3x - 4$$
$$sum = -3$$
$$product = -4$$
$$numbers = 1 \,\&\, -4$$
$$(x+1)(x-4)$$

In the case that a is not 1, we use the box method

Box Method
Just like the box method of multiplication, make a box with four squares. This time, we will fill in the inside first and then figure out what goes on the top and side. The top left term is ax^2, and the bottom right term is c. To figure out the top right and bottom left terms, use the sum and product method. The terms for those boxes will be n_1 and n_2. Once you have all the squares filled in, figure out the GCF of each row or column to determine the terms for the top and side. Then you can just read off the terms and write your factors.

$$6x^2 - 7x - 5$$
$$sum = -7$$
$$product = -30$$
$$numbers = -10 \,\&\, 3$$

$6x^2$	-10x
3x	-5

	3x	-5
2x	$6x^2$	-10x
1	3x	-5

$$(3x-5)(2x+1)$$

Grouping Method
The grouping method is used to factor polynomials with 4 terms. The first step is to group the polynomial into two groups of two terms. Then you use the GCF method to factor each group. If the polynomial can be factored using the grouping method, the binomials in parentheses will match, and you can combine the groups into your final answer.

$$\begin{aligned} &6x^3 - 8x^2 + 15x - 20 \\ &= (6x^3 - 8x^2) + (15x - 20) \\ &= 2x^2(3x - 4) + 5(3x - 4) \\ &= (2x^2 + 5)(3x - 4) \end{aligned}$$

Zero-Product Rule
If the product of 2 or more factors is zero, then one of the factors must be zero.

Solving equations by factoring can help find multiple solutions to an equation. You must begin with an expression that is equal to zero. If you begin with something not equal to zero, subtract terms from both sides until one side of the equation is zero. Now factor the expression. One of these factors must equal zero, so setting each factor equal to zero gives all of the possible solutions.

$$\begin{aligned} x^2 + 8 &= 9x \\ x^2 - 9x + 8 &= 0 \\ (x - 1)(x - 8) &= 0 \end{aligned}$$

$$x - 1 = 0 \text{ OR } x - 8 = 0$$

$$x = 1 \text{ OR } x = 8$$